

A Bayesian Approach to Adult Human Age Estimation from Dental Observations by Johanson's Age Changes

REFERENCE: Lucy, D., Aykroyd, R. G., Pollard, A. M., and Solheim, T., "A Bayesian Approach to Adult Human Age Estimation from Dental Observations by Johanson's Age Changes," *Journal of Forensic Sciences*, JFSCA, Vol. 41, No. 2, March 1996, pp. 189-194.

ABSTRACT: Much of the data which appears in the forensic and archaeological literature is ordinal or categorical. This is particularly true of the age related indicators presented by Gustafson [1] in his method of human adult age estimation using the structural changes in human teeth. This technique is still being modified and elaborated. However, the statistical methods of regression analysis employed by Gustafson and others are not particularly appropriate to this type of data, but are still employed because alternatives have not yet been explored. This paper presents a novel approach based upon the application of Bayes' theorem to ordinal and categorical data, which overcomes many of the problems associated with regression analysis.

KEYWORDS: forensic science, forensic anthropology, Bayes' theorem, age estimation, human identification

Since 1950, when Gustafson first published his seminal paper (1) on adult human age estimation using morphological changes in the structure of teeth, there have been many papers that have adopted and refined Gustafson's basic methodology.

Traditionally, efforts have concentrated on improvements in the scoring system and the variables used to estimate age. Gustafson used six variables: attrition, secondary dentine apposition, periodontal recession, root resorption and root dentine translucency (1). For these, Gustafson awarded a score on a scale of zero to three based upon the visual severity of the change, and employed linear regression to calculate an equation that linked age to the sum of points for all the variables for each tooth. Dalitz (2) divided the age changes observed by Gustafson into five stages. Dalitz used multiple regression and found that cementum build-up and root resorption could be discarded with no effect upon the accuracy or precision of age estimates. Bang and Ramm (3) used only one of the six variables, root dentine translucency. They found that

accurate estimates of age could be made by measuring on a continuous scale the extent of the translucent root zone. They used curvilinear regression to derive for each tooth locus an expression for the relationship between the extent of the translucent zone and age. Johanson (4) used all of Gustafson's age related changes, but scored using half stages, which he demonstrated could be detected. He also used multiple regression to calculate a regression line from which ages for unknown individuals could be estimated. Burns and Maples (5) used a points system based upon Gustafson's but worked on a basis of using separate regression lines for each tooth locus. In addition, they took into account other factors such as sex, age, and whether the individual had a history of periodontal disease. Maples (6) used multiple regression to examine which of the age related changes were least correlated with age the intention being to achieve the same success at estimating age, but with fewer parameters. Maples found that of the age-related changes, root resorption was by far the least related, and should not be used for age estimation. Root dentine transparency and secondary dentine were considered to be the best age indicators and the second molar was said to be the best tooth to use. Kashyap and Koteswara-Rao (7) followed Bang and Ramm in measuring the extent of root dentine translucency, but included secondary dentine, attrition and cementum apposition measured on similar continuous scales. They calculated separate regression lines for each type of tooth locus as a function of age, then calculated the age for an individual by taking a mean of the separate age estimates.

Despite all the improvements made to adult age estimation by modification of the variables used and the way in which they are scored, little attention has been paid to the statistical means by which adult age estimates are derived. With the exception of the work of Bang and Ramm (3) and Kashyap and Koteswara Rao (7) all the techniques outlined rely on the use of points awarded to the extent of the age related change and regression analysis. The use of regression analysis for categorical data may be responsible for some of the problems encountered by those engaged in this field of research and has possibly led to some of the criticisms (8,9) of Gustafson's (1) original work, which focused upon Gustafson's estimate of the error associated with the technique.

The Assumptions of Regression Analysis

There are a number of assumptions about the nature of the data treated by regression analysis. These are:

1. Variables give independent information about age. This means that the age related changes cannot be dependent physically upon each other. The changes can be correlated with each other, and this would be expected given that they all change systematically with age, but increase in one variable

¹Research Student, Department of Archaeological Sciences, University of Bradford, Bradford, West Yorkshire, England.

²Professor, Department of Statistics, School of Mathematics, University of Leeds, Leeds, West Yorkshire, England.

³Professor, Department of Archaeological Sciences, University of Bradford, West Yorkshire, England.

⁴Professor, Department of Oral Pathology and Section for Forensic Odontology, Blindern, Oslo, Norway.

Received for publication 29 May 1995; revised manuscript received 10 Aug. 1995; accepted for publication 11 Aug. 1995.

cannot directly cause change in another. This means we would expect partial correlations between the age changes to approach zero when controlled for age.

2. Variables vary continuously with age. A continuous variable has to be theoretically capable of adopting an infinite number of values, such as the linear measurement of root dentine translucency used by Bang and Ramm (3). However, the seven stages adopted by Johanson (4) for age related variables comes nowhere near to approximating a continuous variable, and is more properly thought of as an ordinal variable.
3. The error distribution about the mean of any variable for a given age is normal, which essentially means that the variables are multivariate normal, or uni-variate normal if only one variable is being used. This means that any estimation of error on an unknown age will be normally distributed about its predictive value. This is an additional constraint that may not be completely fulfilled when the data are examined. A corollary of this assumption is that the predicted variable should be continuous. So, even were some form of inverse regression to be considered appropriate (9), where age change is regressed as the response variable against age change, which is the controlling variable; the age change should be on a continuous scale.
4. When linear regression, either multivariate or univariate, is carried out, there is the assumption that the variable changes linearly with age. This does not apply to curvilinear regression as used by Bang and Ramm (3).
5. When summation methods are used, such as by Gustafson (1), it is assumed that all the variables are contributing the same amount of information about age. This has subsequently been demonstrated to be untrue by Johanson (4).

Of the procedures described, Gustafson (1) implicitly makes all five assumptions, as do Burns and Maples (5). Dalitz (2) and Johanson (4) make the first three assumptions. Maples (6) makes the first four assumptions. Because some, or all of, those assumptions have been made, regression analysis does not inevitably lead to estimated ages that are necessarily inaccurate or imprecise. However, neither is there any rationally justifiable reason for its use in situations that involve categorical or ordinal data. The plain fact is that suitable non-parametric predictive regression techniques have not, as yet, been applied to this field.

Age Estimation and Bayes' Theorem

Alternatives to regression analysis do exist. For some time non-parametric methods have been used in the estimation of the age structure of a population. For example, Konigsberg and Frankenberg (10) describe the potential of an iterative expectation maximization algorithm for the accurate estimation of age structures for human populations. However, the forensic or archaeological scientist is more concerned with the age of individuals in a population and one of the elements used as a basis for Konigsberg and Frankenberg's work can be adapted for estimating the age of individuals. This is the theory of probability called Bayes' theorem. The Bayesian paradigm involves three important concepts: *prior probability*, *likelihood* and *posterior probability* (11; pp. 57–60).

The *prior probability* is the initial assignment of the probability of any hypothesis being true before experimental evidence is considered. In age estimation this would be the probability of an individual belonging to a defined age group given no information (other than that the individual is similar to the reference sample).

This probability is given the notation $P(A_i)$, which is the probability of the individual having an age which falls into age category i .

The *likelihood* is the conditional probability of the observed information, I , given the hypothesis is true and is given the notation $P(I|A_i)$.

The *posterior probability* is the conditional probability of a hypothesis being true given the value of the observed information and in notation is $P(A_i|I)$. This is the probability that an individual belongs to age category i after taking into account both prior information from the reference sample and observed evidence from the indicator variables.

Bayes' theorem states that the posterior probability is proportional to the prior probability multiplied by the likelihood. The constant of proportionality is given by the reciprocal of the sum over all age categories of the product of corresponding prior probabilities and likelihoods. Using the notation described above, and where j refers to all i 's, this can be written:

$$P(A_i|I) = \frac{P(A_i) \times P(I|A_i)}{\sum P(A_j) \times P(I|A_j)} \quad (1)$$

Of course in practice we might record more than one piece of information in the application considered in this paper five measurements were taken. Hence, in general we might consider n measurements and write our observed information as $I = \{I_1, I_2, \dots, I_n\}$. It can be shown that the generalization of Bayes' theorem for this situation is;

$$P(A_i|\{I_1, I_2, \dots, I_n\}) = \frac{P(\{I_1, I_2, \dots, I_n\}|A_i)P(A_i)}{\sum P(\{I_1, I_2, \dots, I_n\}|A_j)P(A_j)} \quad (2)$$

By assuming *conditional independence* of the observed measurements given the age category, Equation 2 can be re-written;

$$P(A_i|\{I_1, I_2, \dots, I_n\}) = \frac{P(I_1|A_i)P(I_2|A_i) \cdots P(I_n|A_i)P(A_i)}{\sum P(I_1|A_j)P(I_2|A_j) \cdots P(I_n|A_j)P(A_j)} \quad (3)$$

All probabilities can then be estimated from the observed data. For ease of computation we may first introduce a quantity $\rho(i)$ which is the numerator in Equation 3, that is;

$$\rho(i) = P(I_1|A_i)P(I_2|A_i) \cdots P(I_n|A_i)P(A_i) \quad (4)$$

The denominator in equation (3) being given by;

$$\sum_{j=1}^n \rho(j) \quad (5)$$

and hence we can recover $P(A_i|\{I_1, I_2, \dots, I_n\})$ using;

$$P(A_i|\{I_1, I_2, \dots, I_n\}) = \frac{\rho(i)}{\sum \rho(j)} \quad (6)$$

Again, in practice we will estimate each of the probabilities in Equation 4 by an appropriate proportion from the reference sample, for example;

$$P(I_1 | A_i) = \frac{n(I_1, A_i)}{n(A_i)} \quad (7)$$

where $n(I_1, A_i)$ is the number of cases in the reference sample with the particular indicator variable I_1 and age group i , and similarly $n(A_i)$ is the total number in age group i . In addition, the total number of cases in the reference sample is N . Using this expression Equation 4 can be re-written;

$$\rho(i) = \frac{n(I_1, A_i)}{n(A_i)} \cdot \frac{n(I_2, A_i)}{n(A_i)} \cdots \frac{n(I_m, A_i)}{n(A_i)} \cdot \frac{n(A_i)}{N} \quad (8)$$

This, along with Equation 5, yields a posterior probability distribution for the age of an individual based upon a reference sample after all observed information has been taken into account. The only assumption made by this method of treating multivariate ordinal data is that all the indicator variables are conditionally independent given age.

An Example of Bayesian Prediction

Data on second maxillary incisors was taken as a subset of data previously published (12–15). Data from left maxillary second incisors and right maxillary second incisors were pooled since previous work (12–15) showed no differences in the rate of development on contralateral teeth. All teeth came from separate individuals, obviating autocorrelation between measurements. The variables used were picked for their strong relationship to chronological age and to avoid dependence of measurement. They were secondary dentine, periodontal recession, apical translucency, root color and the surface roughness of the cementum. All variables were scored on the incremental scale of Johanson (4), with the notation of color estimate and surface roughness having been described by Solheim (16,17).

Table 1 gives a cross tabulation of these data based upon age group and severity of change. For example: the first cell for secondary dentine has a value of zero. This means that there are no people in the reference sample who have a secondary dentine score of zero and who are aged between 11 and 20. The cell to the right has a value of one; there was one person in the reference sample with secondary dentine score of value one who was aged between 11 and 20. The row totals are the number of individuals in each age group in the reference sample.

Were there to be an individual who had the following scores: secondary dentine 1, periodontal recession 2, root dentine translucency 3, color estimate 2, root surface roughness score 2, we can look up from Table 1 suitable values to insert into Equation 8. For example $\rho(21 < \text{age} < 30)$ is calculated by looking at the column which corresponds to the observed value of secondary dentine, in this case 1, and taking that value that corresponds to the 21–30 age group, this value is 4. This is then divided by the total number of individuals from the reference sample who belong to that age group, which is the row total, in this case 7. Again, for periodontal recession we look up the value in the column which corresponds to the observed score, this time the column headed 2, and age group 21–30; this value is 2, which is then divided by the row total, 7. This process is repeated for all variables. The final term in Equation 8 is the number of people in the reference sample in the age group, the row total, divided by the total number in the reference sample, here 71.

$$\rho(21 < \text{age} < 30) = \frac{4}{7} \times \frac{2}{7} \times \frac{2}{7} \times \frac{5}{7} \times \frac{3}{7} \times \frac{7}{71} = 1.41 \times 10^{-3}$$

All these values are then multiplied together to produce $\rho(i)$ for that individual for that age group. A $\rho(i)$ value is then calculated as above for each age group and these are summed to produce the denominator of Equation 5. Each $\rho(i)$ is then divided by this sum to yield posterior probabilities of that individual belonging to each age group.

Age (i)	$\rho(i)$	$P(A_i I_1 \dots I_5)$
11–20	0	0
21–30	1.41×10^{-3}	0.274
31–40	1.23×10^{-3}	0.239
41–50	2.51×10^{-3}	0.487
51–60	0	0
61–70	0	0
71–80	0	0
81–90	0	0
$\Sigma \rho(i) = 5.146 \times 10^{-3}$		

We can therefore have 27% confidence that the individual is between 21 and 30 years of age, a 23% confidence that the individual is between 31 and 40, and a 48% confidence that they are between 41 and 50. There is 0% confidence that this individual belongs to any other age group.

As an illustration the data from 71 maxillary second incisors used in Table 1 had ages estimated for them using both multiple regression and the Bayesian prediction outlined above. As it would be incorrect to make estimates for individuals which also appeared in the reference sample, it was decided to use a jackknife resampling strategy (18). This involves removing each case in turn, calculating a regression line or cross tabulation on the basis of the other cases in the reference sample, and then estimating the age of the separated case. This routine is run for all cases in the sample. So that the results of Bayesian prediction can be compared with parametric multiple regression, the median age was taken as an analogue to the mean suggested by parametric regression. This is the age corresponding to the 50th percentile of the posterior probability distribution, and was calculated by linear interpolation. Likewise upper and lower confidence limits were calculated which were the ages corresponding to the 2.5 and 97.5 percentiles of the area of the probability distribution.

The scores, estimated ages and 95% confidence intervals were calculated by parametric multiple regression and Bayesian prediction for all 71 individuals using routines devised by the authors. The results are presented in Table 2.

The mean of the absolute errors for all cases can be taken as a measure of accuracy. For the Bayesian prediction analysis this was 7.0 years; for parametric multivariate regression the mean of the absolute error was 7.8 years. The average 95% confidence interval for Bayesian prediction was 19.4 years whereas the average 95% confidence interval for parametric regression was 37.9 years.

Discussion

The Bayesian analysis mentioned compares well with parametric multiple regression in this instance, being slightly more accurate and placing predictions within 95% confidence limits, which are on average half as wide as parametric multiple regression. The reason for this marked improvement in the confidence interval is

TABLE 1—Cross-tabulation by age and score for 71 maxillary second incisors.

Age Group	Secondary Dentine Score by Johanson							Row Total
	0	1	2	3	4	5	6	
11-20	0	1	0	0	0	0	0	1
21-30	0	4	0	3	0	0	0	7
31-40	0	3	7	2	1	1	0	14
41-50	0	3	4	2	5	2	0	16
51-60	0	0	2	4	1	1	1	9
61-70	0	0	0	1	3	1	1	6
71-80	0	0	0	4	3	4	4	15
81-90	0	0	0	0	1	1	1	3

Age Group	Root Dentine Translucency Score by Johanson							Row Total
	0	1	2	3	4	5	6	
11-20	1	0	0	0	0	0	0	1
21-30	0	2	2	2	1	0	0	7
31-40	0	0	6	4	4	0	0	14
41-50	0	1	0	9	3	3	0	16
51-60	0	0	0	3	2	4	0	9
61-70	0	0	0	0	1	4	1	6
71-80	0	0	0	0	2	10	3	15
81-90	0	0	0	0	0	2	1	3

Age Group	Periodontal Recession Score by Johanson							Row Total
	0	1	2	3	4	5	6	
11-20	1	0	0	0	0	0	0	1
21-30	1	4	2	0	0	0	0	7
31-40	0	1	5	4	3	0	1	14
41-50	0	0	4	7	3	1	1	16
51-60	0	0	1	6	2	0	0	9
61-70	0	0	1	3	1	1	0	6
71-80	0	0	8	3	4	0	0	15
81-90	0	0	1	1	1	0	0	3

Age Group	Color Estimate Score by Solheim					Row Total
	1	2	3	4	5	
11-20	1	0	0	0	0	1
21-30	2	5	0	0	0	7
31-40	4	8	1	1	0	14
41-50	0	12	4	0	0	16
51-60	0	2	7	0	0	9
61-70	0	0	2	3	1	6
71-80	0	1	3	9	2	15
81-90	0	1	1	1	0	3

Age Group	Root Surface Roughness Estimate Score by Solheim				Row Total
	1	2	3	4	
11-20	0	1	0	0	1
21-30	2	3	2	0	7
31-40	1	7	6	0	14
41-50	0	9	6	1	16
51-60	0	8	1	0	9
61-70	0	1	5	0	6
71-80	0	1	9	5	15
81-90	0	0	3	0	3

TABLE 2—Ages, scores and estimated ages for 71 second maxillary incisors.

Individual ages and scores						Estimated ages and confidence intervals by Bayesian prediction				Estimated ages and confidence intervals by Parametric regression			
Known age	Second- ary dentine	Perio- dental recession	Trans- lucency	Color estimate	Root roughness estimate	Lower 95% con- fidence limit	Esti- mated (median) age	Upper 95% con- fidence limit	Deviation of estimate from real age	Lower 95% con- fidence limit	Esti- mated age	Upper 95% con- fidence limit	Deviation of estimate from real age
17	1	0	0	1	2	21.49	25.95	30.40	8.95	-3.79	16.14	36.06	0.86
22	1	1	2	2	2	21.72	28.29	39.36	6.29	13.76	32.44	51.12	10.44
24	1	1	3	1	2	21.80	29.07	39.62	5.07	13.34	32.22	51.11	8.22
24	1	1	1	2	3	21.49	25.95	30.40	1.95	11.67	31.14	50.62	7.14
27	1	1	4	2	2	31.49	35.95	40.40	8.95	25.29	43.98	62.68	16.98
28	3	0	3	1	1	21.75	28.62	39.49	0.62	15.33	35.31	55.30	7.31
30	3	2	1	2	1	21.49	25.95	30.40	4.06	7.62	27.25	46.87	2.75
30	3	2	2	2	3	27.50	35.54	40.36	5.54	22.64	41.25	59.86	11.25
31	1	2	3	1	3	21.72	28.29	39.36	2.71	16.01	35.04	54.08	4.04
31	5	4	2	2	2	31.99	41.17	58.48	10.17	23.42	42.81	62.20	11.81
32	3	2	3	2	2	22.58	43.30	56.83	11.30	24.03	42.28	60.54	10.28
34	2	2	4	2	3	31.79	38.97	49.79	4.97	30.71	48.97	67.23	14.97
34	2	6	2	2	2	31.56	36.69	47.09	2.69	10.70	30.85	51.00	3.15
35	2	3	2	1	2	31.49	35.95	40.40	0.95	8.73	27.36	46.00	7.64
35	2	4	2	2	3	31.49	35.95	40.40	0.95	17.54	36.57	55.60	1.57
36	1	2	2	2	3	21.85	29.59	39.73	6.41	15.83	34.72	53.61	1.28
37	2	1	4	3	1	22.23	51.23	59.93	14.23	28.98	48.69	68.40	11.69
37	3	3	4	2	2	38.47	52.47	60.08	15.47	28.68	47.02	65.36	10.02
38	1	3	3	1	2	31.49	35.95	40.40	2.06	10.82	29.62	48.42	8.38
39	2	2	4	2	2	31.90	40.10	51.01	1.10	26.14	44.72	63.30	5.72
39	2	4	2	4	3	31.49	35.95	40.40	3.06	29.23	49.13	69.02	10.13
39	4	3	3	1	3	34.80	45.60	54.34	6.60	23.72	42.96	62.20	3.96
41	1	2	1	2	2	21.49	25.95	30.40	15.06	5.74	24.29	42.84	16.71
41	1	2	3	2	3	22.43	34.99	49.06	6.01	21.27	40.03	58.80	0.97
43	4	2	3	2	3	33.57	44.80	50.37	1.80	30.25	48.89	67.52	5.89
43	2	6	5	3	2	49.16	55.65	60.37	12.65	35.08	55.15	75.23	12.15
44	4	3	3	3	2	42.02	52.92	60.10	8.92	31.35	49.90	68.45	5.90
44	1	3	3	2	3	32.35	43.10	50.12	0.90	20.41	39.16	57.92	4.84
44	3	4	4	2	4	71.49	75.95	80.40	31.95	35.73	54.99	74.25	10.99
46	2	3	3	2	3	32.08	41.86	50.62	4.14	23.62	42.14	60.66	3.86
47	4	3	4	3	2	42.83	55.42	66.42	8.42	36.81	55.16	73.50	8.16
47	3	2	3	2	2	22.34	35.61	57.44	11.39	23.42	41.80	60.19	5.20
47	2	3	3	2	2	32.51	44.45	58.07	2.55	19.95	38.26	56.57	8.74
47	4	5	5	3	3	61.49	65.95	70.40	18.95	45.59	63.96	82.34	16.96
48	2	3	3	2	2	32.51	44.45	58.07	3.55	19.95	38.23	56.51	9.77
48	5	4	5	2	2	43.11	55.02	76.42	7.02	38.97	58.08	77.18	10.08
48	4	4	3	2	2	35.05	45.82	56.04	2.18	24.51	43.25	62.00	4.75
50	5	3	4	2	3	32.00	41.24	75.80	8.76	37.73	56.50	75.27	6.50
52	3	3	3	3	2	42.89	54.74	60.28	2.74	28.11	46.57	65.04	5.43
53	2	3	3	2	2	32.80	44.20	50.60	8.80	20.00	38.06	56.12	14.94
55	3	2	4	3	2	32.72	45.08	77.24	9.92	34.04	52.57	71.10	2.43
57	2	3	5	2	2	41.58	46.86	57.73	10.14	29.20	48.21	67.21	8.79
57	5	3	4	3	3	41.21	65.50	79.12	8.50	43.16	61.66	80.15	4.66
58	6	3	5	3	2	61.70	68.06	79.24	10.06	47.66	66.59	85.53	8.59
58	3	4	5	3	2	46.95	56.31	75.31	1.69	37.82	56.64	75.46	1.36
59	4	3	5	3	2	42.00	61.22	70.69	2.22	41.61	60.22	78.82	1.22
59	3	4	3	3	2	41.13	53.20	60.13	5.80	27.07	45.43	63.79	13.57
65	3	2	5	3	3	52.69	75.33	80.34	10.33	42.97	61.55	80.12	3.45
66	5	5	4	5	2	41.63	47.39	78.70	18.61	47.54	67.66	87.78	1.66
66	4	3	6	4	3	71.49	75.95	80.40	9.94	56.60	75.19	93.77	9.19
68	4	3	5	4	3	61.91	70.15	79.82	2.15	50.89	69.41	57.92	1.41
68	6	4	5	4	3	71.49	75.95	80.40	7.94	56.13	74.84	93.54	6.84
68	4	3	5	3	3	43.48	65.18	79.07	2.82	45.43	63.81	82.18	4.19
72	5	4	5	4	3	63.84	74.90	80.30	2.90	52.97	71.54	90.12	0.46
73	3	2	5	3	3	50.38	73.39	80.15	0.39	42.79	61.15	79.50	11.85
73	5	2	6	4	4	71.49	75.95	80.40	2.94	63.83	82.83	101.84	9.83
74	3	4	6	5	4	71.49	75.95	80.40	1.94	61.90	81.68	101.47	7.68
74	6	3	5	2	3	51.49	55.95	60.40	18.06	43.56	62.73	81.91	11.27
74	4	4	5	4	3	61.88	69.86	79.78	4.14	49.97	68.50	87.02	5.50
75	4	2	5	5	2	61.49	65.95	70.40	9.06	51.08	70.94	90.81	4.06
75	3	3	5	4	4	71.49	75.95	80.40	0.94	50.70	69.78	88.86	5.22
76	6	2	5	4	4	71.49	75.95	80.40	0.06	60.48	79.83	99.18	3.83
76	5	2	4	4	3	40.29	74.29	80.24	1.71	47.99	66.76	85.54	9.24
77	6	2	4	3	3	56.68	72.61	80.07	4.39	44.83	63.61	82.39	13.39
79	3	2	5	3	3	50.38	73.39	80.15	5.61	42.82	60.85	78.88	18.15
79	5	2	5	4	3	66.19	75.42	80.35	3.58	53.70	72.36	91.03	6.64
79	4	4	5	4	3	61.88	69.86	79.78	9.14	49.88	68.26	86.64	10.74
79	6	3	6	4	4	71.49	75.95	80.40	3.06	65.53	84.55	103.57	5.55
80	5	2	5	3	3	52.69	74.46	80.26	5.54	48.32	66.65	84.99	13.35
80	6	4	6	4	3	64.31	75.07	80.32	4.93	60.84	79.63	98.43	0.37
86	4	3	5	2	3	41.57	46.82	75.74	39.18	39.24	56.52	73.80	29.48

that in cases where the probability is not evenly distributed about a mean then the parametric multiple regression has to adopt a wider confidence interval. The Bayesian prediction outlined here makes no such assumption about the distribution of probability about the age estimate so it can predict age ranges which vary according to the empirical posterior probability distribution, reflecting a more realistic case-by-case approach to the estimation of error.

Although in the case of the 71 maxillary incisors above, where median estimates of age have been calculated for comparative discussion, age estimates using the Bayesian predictive model are given as a probability assigned to each arbitrarily defined age group, not as a mean estimate of age with an evenly distributed error around that mean. This approach has the advantage that in cases which are genuinely ambiguous, for instance if an individual has suffered heavy periodontal disease, or has anomalously high root dentine translucency (3; p. 27), then the probability distribution will be bi-modal. These cases can be picked out as unreliable and subjected to further examination. In similar ambiguous cases parametric multiple regression will assign a mean age, which lies somewhere between the two ages suggested by the data giving no obvious clue that something may be amiss.

The problem with the Bayesian prediction analysis given above is that only 79% of cases does the known age fall into the 95% confidence intervals. Parametric regression places over 99% of known ages within the 95% confidence interval. This suggests confidence interval may be slightly too small, which could be a feature of this particular data set. However, one drawback with this form of analysis is that an accurate estimate of the distribution of likelihoods can only be obtained when the reference sample is large. The cross-tabulation shown in Table 1 has many cells that are empty, or have a small number of entries in them.

Conclusions

It is evident that the non-parametric method of analyzing ordinal data as used here in connection with points based dental methods of age estimation is more robust and appropriate than the traditional application of regression models to categorical/ordinal data for which there is no statistically rational justification. This is because valid error estimates on regression models require normally distributed variables. The ordinal/categorical measurements used in Gustafson's aging technique do not approximate well to a continuous variable such as age, and regression techniques can entail a loss of information which obscures the real probability distribution of any predicted value. With the Bayesian prediction procedure the output is a probability distribution, not a mean with an artificial standard deviation, which means that there is little loss of predictive information and that cases which genuinely carry ambiguities (such as bimodal probability distributions) can be singled out for further examination.

Because none of the assumptions inherent in conventional regression techniques are required by the Bayesian approach, the predictions are better than those made by conventional regression techniques in that their average absolute error can be lower, and the 95% confidence intervals are smaller.

Provided the assumption that variables are conditionally independent is not violated, an assumption that is also inherent in all

regression models, this method should prove to be capable of further development into a powerful statistical technique appropriate for use in other fields where the forensic and archaeological scientist has to work with empirically derived categorical and/or ordinal data. An obvious further extension is into the area of skeletal indicators of age at death, possibly combined with the best of the dental indicators. Further work may also consist of devising means by which more accurate estimates of likelihoods can be obtained in situations where the reference samples may be small. We believe that the Bayesian approach has unlimited potential in this crucial area of interest to both archaeological and forensic scientists.

Acknowledgments

The authors would like to thank Dr. P. Rhodes of the Computer Science Department, University of Bradford, for commenting on an earlier draft of this paper. One of the authors (DL) gratefully acknowledges the support of the British Academy for the provision of a research studentship.

References

- (1) Gustafson G. Age determination on teeth. *J Am Dent Assoc* 1950;41:45-54.
- (2) Dalitz GD. Age determination of adult human remains by teeth examination. *J Forensic Sci Soc* 1962;3:11-21.
- (3) Bang G., Ramm E. Determination of age in humans from root dentine transparency. *Acta Odontol Scand* 1970;28:3-35.
- (4) Johanson G. Age determinations from human teeth. *Odontologisk Revy* 1971;22:supplement 2.
- (5) Burns KR, Maples WR. Estimation of age from individual adult teeth. *J Forensic Sci* 1976;21:343-56.
- (6) Maples WR. An improved technique using dental histology for the estimation of adult age. *J Forensic Sci* 1978;23:764-70.
- (7) Kashyap VK, Koteswara-Rao NR. A modified Gustafson method of age estimation from teeth. *Forensic Sci Int* 1990;47:237-47.
- (8) Maples WR, Rice PM. Some difficulties in the Gustafson dental age estimations. *J Forensic Sci* 1979;24:168-72.
- (9) Lucy D., Pollard AM. Further comments on the estimation of error associated with the Gustafson dental age estimation method. *J Forensic Sci* 1995;40(2):222-27.
- (10) Konigsberg LW, Frankenberg SR. Estimation of age structure in anthropological demography. *Am J Physical Anthropol* 1992;89:235-56.
- (11) Phillips LD. *Baysian Statistics for Social Scientists*. Thomas Nelson and Sons Ltd., London, 1973.
- (12) Solheim T. Dental root transparency as an indication of age. *J Dental Res* 1989;97:189-97.
- (13) Solheim T. Dental cementum apposition as an indicator of age. *Scandinav J Dental Res* 1990;98(6):510-19.
- (14) Solheim T. Amount of secondary dentin as an indicator of age. *Scandinav J Dental Res* 1992;100(4):193-99.
- (15) Solheim T. Dental root surface structure as an indicator of age. *J Forensic Odonto-stomatol* 1993;11:9-21.
- (16) Solheim T. Dental color as an indicator of age. *Gerodontology* 1988;4:114-18.
- (17) Solheim T. Recession of the periodontal ligament as an indicator of age. *Int J Forensic Odont-stomatol* 1993;10:32-42.
- (18) Efron B. *The Jackknife, the Bootstrap, and Other Resampling Plans*. Society for Industrial and Applied Mathematics. Philadelphia, 1982.

Address requests for reprints or additional information to David Lucy
Dept. of Archaeological Sciences
University of Bradford
Bradford, West Yorkshire
England BD7 10P